# Numerical Treatment of the MHD Convective Heat and Mass Transfer in an Electrically Conducting Fluid over an Infinite Solid Surface in Presence of the Internal Heat Generation

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Numerical solutions of a set of non-linear partial differential equations are investigated. We obtained the velocity distribution of a conducting fluid flowing over an infinite solid surface in the presence of an uniform magnetic field and internal heat generation. The temperature and concentration distributions of the fluid are studied as well as the skin-friction, rate of mass transfer and local wall heat flux. The effect of the parameters of the problem on these distributions is illustrated graphically.

*Key words:* Magnetohydrodynamic; Thermal Diffusivity; Electrical Conductivity; Heat Flux; Finite Difference Technique.

## Introduction

Convective flow with heat and mass transfer on a solid surface is occurring, for example, if materials are manufactured by an extrusion processes, or if heat-treated materials travel between a feed roll and a wind-up roll. The convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream was investigated by Vajravelu and Hadjinicolau [1]. Acharya *et al.* [2] discussed the heat and mass transfer over an accelerating surface with a heat source in presence of suction and blowing. Eldabe and Mona [3] studied the heat and mass transfer in a hydromagnetic flow of a non-Newtonian fluid with a heat source over an accelerating surface through a porous medium. Free convective flow through a porous medium bounded by an infinite, porous, vertical plate whose temperature fluctuates harmonically with time was studied in [4]. In the present work the unsteady flow of an electrically conducting fluid with magnetohydrodynamic convective heat and mass transfer over an infinite solid surface is studied. The system is stressed by a uniform transverse magnetic field as well as internal heat generation. Numerical solutions are obtained for the velocity, temperature and mass distribution as well as the skin-friction and the heat and mass transfer.

## 1. Governing Equations

*The continuity equation:*

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \tag{1}$$

*The momentum equation:*

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \tag{2}$$

$$= \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_0 B_0^2}{\rho}u + g\beta_0(T - T_\infty) + g\beta_1(C - C_\infty),$$

*The energy equation:*

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + Q(T - T_\infty), \tag{3}$$

*The equation for species concentration:*

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} \tag{4}$$

The coordinates $(x, y)$ are measured along the semi-infinite plate and normal to it, respectively, with the origin at the leading edge. The X-axis is parallel to the direction of gravity, but directed upward. The magnetic field is in the Y-direction.

$u$ and $v$ are the velocity components in the X- and Y-directions respectively, $\nu$ is the fluid kinematic viscosity, $T$ the temperature, $C$ the concentration, $\rho$ the density, $\beta_0$ the temperature coefficient of volumetric expansion, $\beta_1$ the concentration coefficient of volumetric expansion, $g$ the gravitational acceleration, $\sigma_0$ the electrical conductivity, $B_0$ the strength of a uniform magnetic field, $Q$ the volumetric rate of heat generation, $\alpha$ the thermal diffusivity and $D_m$ the chemical molecular diffusivity.

The initial and boundary conditions to the problem are:

$$\begin{aligned} &t \le 0: u = 0, v = 0, T = T_\infty, C = C_\infty, \\ &t > 0: u = 0, v = 0, T = T_\infty, C = C_\infty \text{ at } x = 0, \\ &\quad u = 0, v = 0, T = T_w, C = C_w \text{ at } y = 0, \\ &\quad u = 0, T \to T_\infty, C \to C_\infty \text{ as } y \to \infty. \end{aligned} \tag{5}$$

Here $t$ is the time, $T$ the temperature of the fluid in the boundary layer, $T_\infty$ the temperature of the fluid far away from the plate, $T_w$ the plate temperature, $C$ the species concentration in the fluid near the plate, $C_\infty$ the concentration of the fluid far away from the plate, and $C_w$ the concentration of the species at the plate.

We introduce the following dimensionless variables:

$$\begin{aligned} &x^* = \frac{x}{L}, y^* = \frac{y}{L}, t^* = \frac{Ut}{L}, u^* = \frac{u}{U}, \\ &v^* = \frac{v}{U}, \nu^* = \frac{\nu}{UL}, \alpha^* = \frac{\alpha}{UL}, Q^* = \frac{QL}{U}, \\ &D_m^* = \frac{D_m}{UL}, \tau = \frac{T - T_\infty}{T_w - T_\infty}, \theta = \frac{C - C_\infty}{C_w - C_\infty}, \end{aligned} \tag{6}$$

where $L$ is a suitable length and $U$ is a typical velocity.

By using (6), the system (1)–(4) with the boundary conditions (5), after dropping the star mark, can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{7}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - Mu + Gr_t\tau + Gr_m\theta, \tag{8}$$

$$\frac{\partial \tau}{\partial t} + u\frac{\partial \tau}{\partial x} + v\frac{\partial \tau}{\partial y} = \alpha\frac{\partial^2 \tau}{\partial y^2} + Q\tau, \tag{9}$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} \tag{10}$$

with the initial and boundary conditions

$$\begin{aligned} &t \le 0: u = 0, v = 0, \tau = 0, \theta = 0, \\ &t > 0: u = 0, v = 0, \tau = 0, \theta = 0 \text{ at } x = 0, \\ &\quad u = 0, v = 0, \tau = 1, \theta = 1 \text{ at } y = 0, \\ &\quad u = 0, \tau \to 0, \theta \to 0 \text{ as } y \to \infty. \end{aligned} \tag{11}$$

Here, the dimensionless parameters are defined as

$M = \dfrac{\sigma_0 B_0^2 U}{\rho}$ (magnetic parameter);

$Gr_t = \dfrac{g\beta_0(T_w - T_\infty)L}{U^2}$ (local temperature Grashof);

$Gr_m = \dfrac{g\beta_1(C_w - C_\infty)L}{U^2}$ (local mass Grashof number).

Solving the system of partial differential equations numerically, using the finite difference technique and (7–10) are finds

$$\frac{u'_{i,j} - u'_{i-1,j}}{\Delta x} + \frac{v'_{i,j} - v'_{i,j-1}}{\Delta y} = 0, \tag{12}$$

$$\frac{u'_{i,j} - u_{i,j}}{\Delta t} + u_{i,j}\frac{u_{i,j} - u_{i-1,j}}{\Delta x} + v_{i,j}\frac{u_{i,j+1} - u_{i,j}}{\Delta y} = \nu\frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{(\Delta y)^2} - Mu_{i,j} + Gr_t\tau'_{i,j} + Gr_m\theta'_{i,j}, \tag{13}$$

$$\frac{\tau'_{i,j} - \tau_{i,j}}{\Delta t} + u_{i,j}\frac{\tau_{i,j} - \tau_{i-1,j}}{\Delta x} + v_{i,j}\frac{\tau_{i,j+1} - \tau_{i,j}}{\Delta y} = \alpha\frac{\tau_{i,j+1} - 2\tau_{i,j} + \tau_{i,j-1}}{(\Delta y)^2} + Q\tau'_{i,j}, \tag{14}$$

$$\frac{\theta'_{i,j} - \theta_{i,j}}{\Delta t} + u_{i,j}\frac{\theta_{i,j} - \theta_{i-1,j}}{\Delta x} + v_{i,j}\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta y} = D_m\frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{(\Delta y)^2} \tag{15}$$

with the new initial and boundary conditions

$$\begin{aligned} &t \le 0: u_{i,j} = 0, v_{i,j} = 0, \tau_{i,j} = 0, \theta_{i,j} = 0, \\ &t > 0: u_{i,j} = 0, v_{i,j} = 0, \tau_{i,j} = 0, \theta_{i,j} = 0 \text{ at } x = 0, \\ &\quad u_{i,j} = 0, v_{i,j} = 0, \tau_{i,j} = 1, \theta_{i,j} = 1 \text{ at } y = 0, \\ &\quad u_{i,j} = 0, \tau_{i,j} \to 0, \theta_{i,j} \to 0 \text{ as } y \to \infty. \end{aligned} \tag{16}$$

## 2. Consistency of the Finite Difference Scheme

The consistency of a finite difference procedure means that the approximate solution of a partial differential equation is not the approximate solution of any other partial differential equation. The consistency is measured in terms of the difference between a differential equation and a difference equation. Here, we can write

$$
\begin{aligned}
\frac{\partial u}{\partial t} &\approx \frac{u'_{i,j}-u_{i,j}}{\Delta t}+O(\Delta t),\\
\frac{\partial u}{\partial x} &\approx \frac{u_{i,j}-u_{i-1,j}}{\Delta x}+O(\Delta x),\\
\frac{\partial u}{\partial y} &\approx \frac{u_{i,j+1}-u_{i,j}}{\Delta y}+O(\Delta y),\\
\frac{\partial \tau}{\partial t} &\approx \frac{\tau'_{i,j}-\tau_{i,j}}{\Delta t}+O(\Delta t),\\
\frac{\partial \tau}{\partial x} &\approx \frac{\tau_{i,j}-\tau_{i-1,j}}{\Delta x}+O(\Delta x),\\
\frac{\partial \tau}{\partial t} &\approx \frac{\tau_{i,j+1}-\tau_{i,j}}{\Delta y}+O(\Delta y),\\
\frac{\partial \theta}{\partial} &\approx \frac{\theta'_{i,j}-\theta_{i,j}}{\Delta t}+O(\Delta t),\\
\frac{\partial \theta}{\partial x} &\approx \frac{\theta_{i,j}-\theta_{i-1,j}}{\Delta x}+O(\Delta x),\\
\frac{\partial \theta}{\partial y} &\approx \frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta y}+O(\Delta y),\\
\frac{\partial^2 u}{\partial y^2} &\approx \frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{(\Delta y)^2}+O(\Delta y)^2,\\
\frac{\partial^2 \tau}{\partial y^2} &\approx \frac{\tau_{i,j+1}-2\tau_{i,j}+\tau_{i,j-1}}{(\Delta y)^2}+O(\Delta y)^2,\\
\frac{\partial^2 \theta}{\partial y^2} &\approx \frac{\theta_{i,j+1}-2\theta_{i,j}+\theta_{i,j-1}}{(\Delta y)^2}+O(\Delta y)^2.
\end{aligned}
$$

For the consistency of (13) we estimate

$$
\begin{aligned}
&\Big\{\frac{u'_{i,j}-u_{i,j}}{\Delta t}+u_{i,j}\frac{u_{i,j}-u_{i-1,j}}{\Delta x}+v_{i,j}\frac{u_{i,j+1}-u_{i,j}}{\Delta y}\\
&\quad -\nu\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{(\Delta y)^2}+Mu_{i,j}-Gr_t\tau'_{i,j}-Gr_m\theta'_{i,j}\Big\}\\
&-\Big\{\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-\nu\frac{\partial^2 u}{\partial y^2}+Mu-Gr_t\tau-Gr_m\theta\Big\}_{i,j}\\
&=O(\Delta t)+u_{i,j}O(\Delta x)+v_{i,j}O(\Delta y)+O(\Delta y)^2, \qquad (17)
\end{aligned}
$$

and for the consistency of (14) we estimate

$$
\begin{aligned}
&\Big\{\frac{\tau'_{i,j}-\tau_{i,j}}{\Delta t}+u_{i,j}\frac{\tau_{i,j}-\tau_{i-1,j}}{x}+v_{i,j}\frac{\tau_{i,j+1}-\tau_{i,j}}{\Delta y}\\
&\quad -\alpha\frac{\tau_{i,j+1}-2\tau_{i,j}+\tau_{i,j-1}}{(\Delta y)^2}-Q\tau'_{i,j}\Big\}\\
&-\Big\{\frac{\partial \tau}{\partial t}+u\frac{\partial \tau}{\partial x}+v\frac{\partial \tau}{\partial y}-\alpha\frac{\partial^2 \tau}{\partial y^2}-Q\tau\Big\}_{i,j}\\
&=O(\Delta t)+u_{i,j}O(\Delta x)+v_{i,j}O(\Delta y)+O(\Delta y)^2. \qquad (18)
\end{aligned}
$$

Similarly with respect to (15):

$$
\begin{aligned}
&\Big\{\frac{\theta'_{i,j}-\theta_{i,j}}{\Delta t}+u_{i,j}\frac{\theta_{i,j}-\theta_{i-1,j}}{x}+v_{i,j}\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta y}\\
&\quad -D_m\frac{\theta_{i,j+1}-2\theta_{i,j}+\theta_{i,j-1}}{(\Delta y)^2}\Big\}\\
&-\Big\{\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}-D_m\frac{\partial^2 \theta}{\partial y^2}\Big\}_{i,j}\\
&=O(\Delta t)+u_{i,j}O(\Delta x)+v_{i,j}O(\Delta y)+O(\Delta y)^2. \qquad (19)
\end{aligned}
$$

Here the R.H.S's of (17) – (19) represent truncation errors which go to zero as $\Delta t \to 0$ with $\Delta x \to 0$ and $\Delta y \to 0$. Hence, our explicit scheme is consistent.

## 3. Stability Condition of the Scheme

We present here a treatment for determining the stability conditions of the finite difference scheme based on [5]. Since an explicit procedure is used, we intend to investigate the longest time-step consistent with stability. The general terms of the Fourier expansion for $u, \tau$ and $\theta$ at a time arbitrarily chosen are all $e^{ipx}e^{iqy}$ apart from a constant with $i=\sqrt{-1}$. At a later time $t$, those terms will become

$$u=F(t)e^{ipx}e^{iqy},\ \tau=G(t)e^{ipx}e^{iqy},\ \theta=H(t)e^{ipx}e^{iqy},$$

and we can write

$$
\begin{aligned}
u_{i,j} &= F(t)e^{ipx}e^{iqy},\\
u'_{i,j} &= F'(t)e^{ipx}e^{iqy},\\
u_{i-1,j} &= F(t)e^{ip(x-\Delta x)}e^{iqy},\\
u_{i+1,j} &= F(t)e^{ip(x+\Delta x)}e^{iqy},\\
u_{i,j-1} &= F(t)e^{ipx}e^{iq(y-\Delta y)},\\
u_{i,j+1} &= F(t)e^{ipx}e^{iq(y+\Delta y)}.
\end{aligned}
$$

Similarly we can write

$$\tau_{i,j} = G(t)e^{ipx}e^{iqy},$$
$$\tau'_{i,j} = G'(t)e^{ipx}e^{iqy},$$
$$\tau_{i-1,j} = G(t)e^{ip(x-\Delta x)}e^{iqy},$$
$$\tau_{i+1,j} = G(t)e^{ip(x+\Delta x)}e^{iqy},$$
$$\tau_{i,j-1} = G(t)e^{ipx}e^{iq(y-\Delta y)},$$
$$\tau_{i,j+1} = G(t)e^{ipx}e^{iq(y+\Delta y)}.$$

and

$$\theta_{i,j} = H(t)e^{ipx}e^{iqy},$$
$$\theta'_{i,j} = H'(t)e^{ipx}e^{iqy},$$
$$\theta_{i-1,j} = H(t)e^{ip(x-\Delta x)}e^{iqy},$$
$$\theta_{i+1,j} = H(t)e^{ip(x+\Delta x)}e^{iqy},$$
$$\theta_{i,j-1} = H(t)e^{ipx}e^{iq(y-\Delta y)},$$
$$\theta_{i,j+1} = H(t)e^{ipx}e^{iq(y+\Delta y)}.$$

Substituting in (13) – (15), regarding the coefficients $u$ and $v$ as constants over any one time-step and denoting the values of $F$, $G$ and $H$ after the time-step by $F'$, $G'$ and $H'$, respectively, (13) – (15) can be written in the form

$$F' = F - F\left[u\frac{\Delta t}{\Delta x}(1-e^{-ip\Delta x}) + v\frac{\Delta t}{\Delta y}(e^{iq\Delta y}-1) - v\frac{\Delta t}{(\Delta y)^2}(e^{iq\Delta y}-2+e^{-iq\Delta y})\right]Gr_t\Delta t G' + Gr_m\Delta t H', \tag{20}$$

$$G' = \frac{G}{1-Q}\left\{1-u\frac{\Delta t}{\Delta x}(1-e^{-ip\Delta x}) - v\frac{\Delta t}{\Delta y}(e^{iq\Delta y}-1) + \alpha\frac{\Delta t}{(\Delta y)^2}(e^{iq\Delta y}-2+e^{-iq\Delta y})\right\}, \tag{21}$$

$$H' = H\left\{1-u\frac{\Delta t}{\Delta x}(1-e^{-ip\Delta x}) - v\frac{\Delta t}{\Delta y}(e^{iq\Delta y}-1) + D_m\frac{\Delta t}{(\Delta y)^2}(e^{iq\Delta y}-2+e^{-iq\Delta y})\right\}. \tag{22}$$

Consider

$$\alpha_1 = 1-u\frac{\Delta t}{\Delta x}(1-e^{-ip\Delta x}) - v\frac{\Delta t}{\Delta y}(e^{iq\Delta y}-1),$$

$$\alpha_2 = \frac{\Delta t}{(\Delta y)^2}(e^{iq\Delta y}-2+e^{-iq\Delta y}).$$

Equations (20) – (22) may be written as

$$F' = A_1F + A_2G + A_3H, \tag{23}$$
$$G' = A_4G, \tag{24}$$
$$H' = A_5H, \tag{25}$$

where

$$A_1 = \alpha_1 + v\alpha_2,$$
$$A_2 = Gr_t\Delta t A_4,$$
$$A_3 = Gr_m\Delta t A_5,$$
$$A_4 = \frac{\alpha_1 + \alpha\alpha_2}{1-Q},$$
$$A_5 = \alpha_1 + D_m\alpha_2.$$

Here $\alpha_1, \alpha_2, A_1, A_2, A_3, A_4$ and $A_5$ are considered to be constants over any one time-step. Equations (23) – (25) can be written in the matrix form

$$\begin{pmatrix} F' \\ G' \\ H' \end{pmatrix} = \begin{pmatrix} A_1 & A_2 & A_3 \\ 0 & A_4 & 0 \\ 0 & 0 & A_5 \end{pmatrix} \begin{pmatrix} F \\ G \\ H \end{pmatrix} \tag{26}$$

i. e.

$$\zeta' = W\zeta$$

where $\zeta$ is the column vector with the elements $F$, $G$ and $H$.

Now, for stability the modulus of each of the eigenvalues of the amplification matrix $W$ must not exceed unity. The eigenvalues of $W$ are

$$\lambda_1 = A_1, \quad \lambda_2 = A_4, \quad \lambda_3 = A_5.$$

Assume that $u$ is everywhere non-negative, $v$ is everywhere non-positive and let

$$a_0 = u\frac{\Delta t}{\Delta x}, \quad a_1 = |v|\frac{\Delta t}{\Delta y}, \quad \text{and} \quad a_2 = \frac{\Delta t}{(\Delta y)^2},$$

hence

$$A_1 = 1 - a_0 - a_1 - 2a_2v + a_0e^{-ip\Delta x} + a_1e^{iq\Delta y} + va_2\left[e^{iq\Delta y} + e^{-iq\Delta y}\right],$$

where $a_0$, $a_1$ and $a_2$ are all real and non-negative. The maximum value of $|A_1|$ occurs when $p\Delta x = r_1\pi$ and $q\Delta y = r_2\pi$ where $r_1$ and $r_2$ are +ve integers.

Fig. 1.

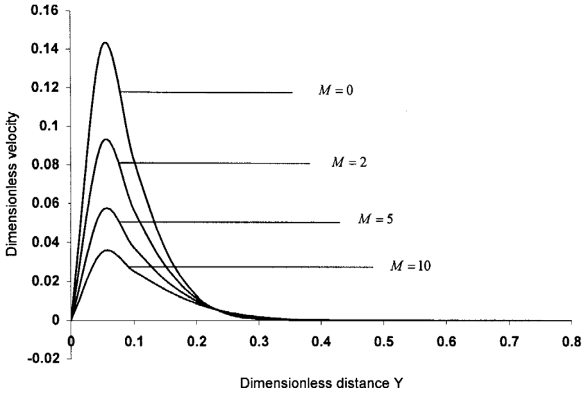



Fig. 2.

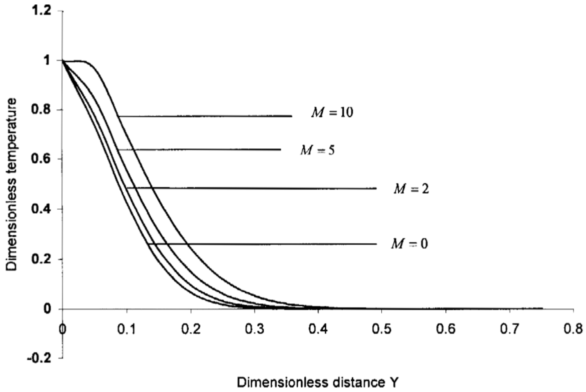
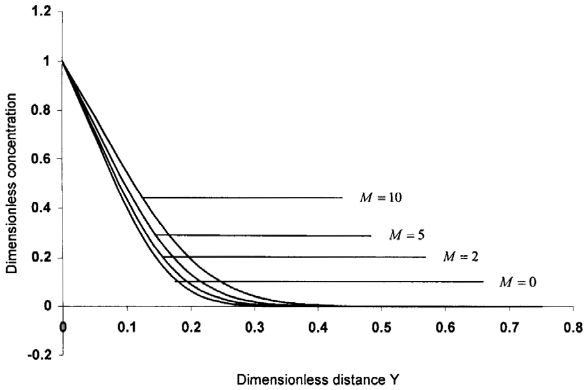



Fig. 3.




Figs. 1 – 3 show that the velocity decreases with increasing magnetic parameter $M$, while the temperature and concentration increase with increasing $M$.

Fig. 4.

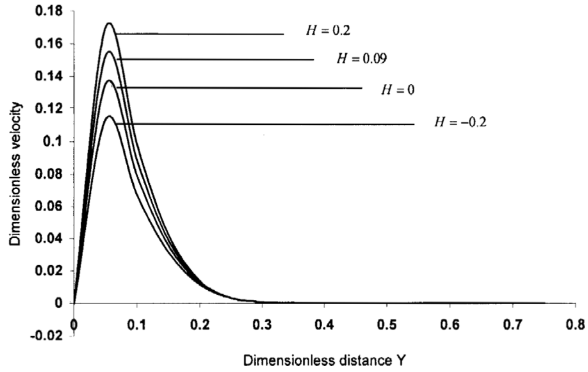



Fig. 5.

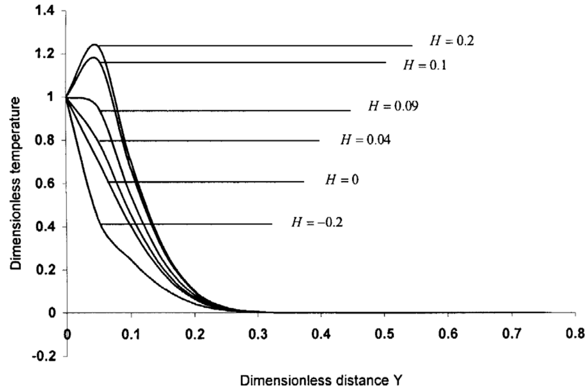
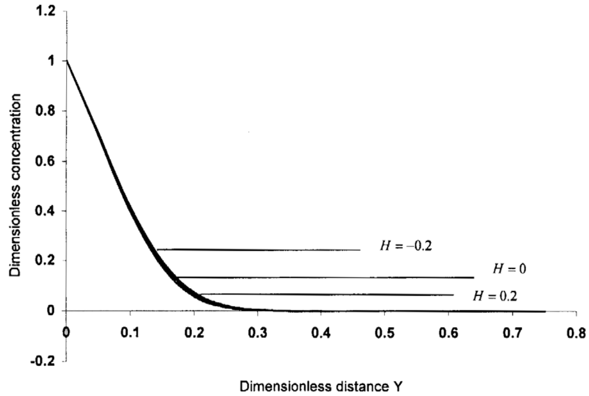



Fig. 6.




Figs. 4 – 6 show that the velocity and temperature increase with increasing heat parameter $H$, while the concentration decreases with increasing $H$.

For sufficiently large $\Delta t$, $|A_1|$ is maximum when $r_1$ and $r_2$ are odd integers. In this case we have

$$A_1 = 1 - 2(a_0 + a_1 + 2a_2 v).$$

To satisfy $|\lambda_1| \le 1$, the most negative allowable value is $A_1 = -1$, hence the stability condition can be written as

$$-1 \le 1 - 2(a_0 + a_1 + 2a_2 v),$$

Fig. 7.

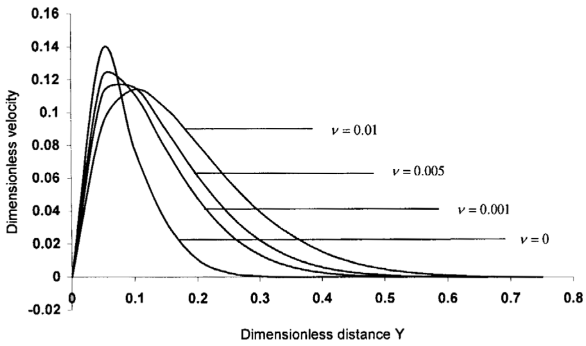



Fig. 8.

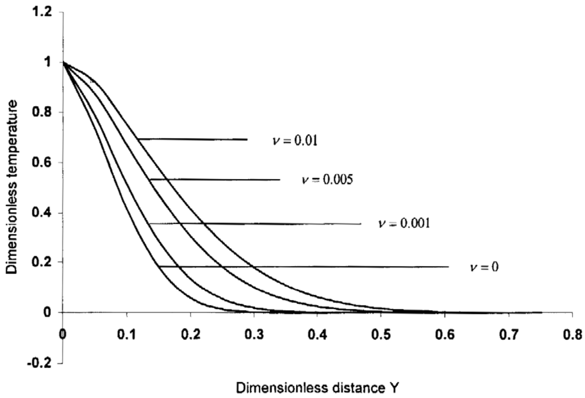
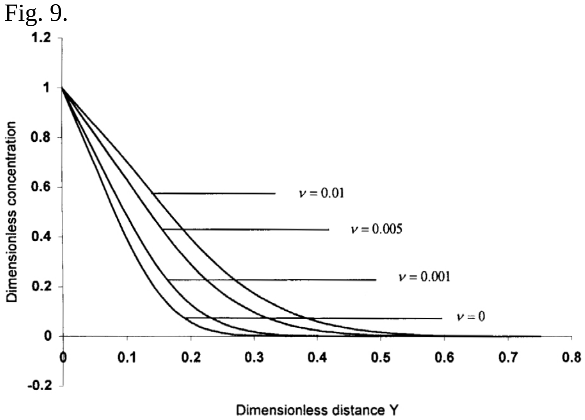



Fig. 9.




Figs. 7 – 9 show that the velocity decreases with increasing $v$, while everywhere the temperature and concentration increase with increasing $v$.

Fig. 10.

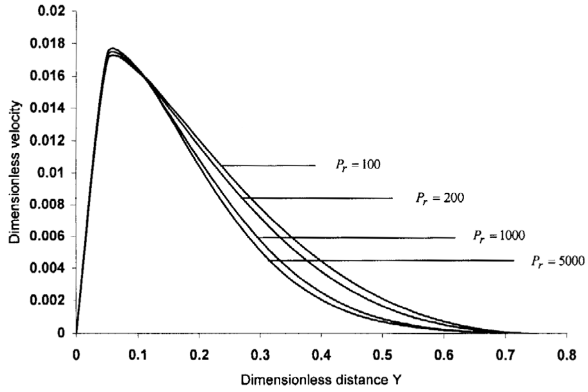



Fig. 11.

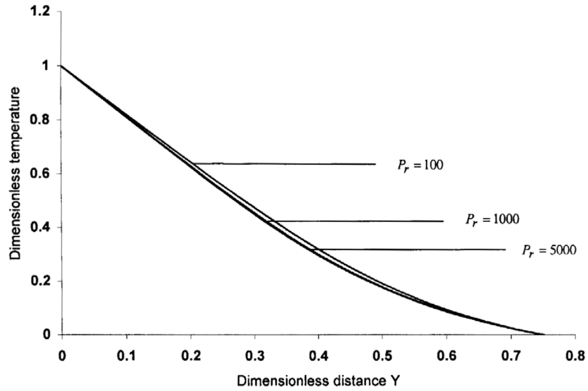
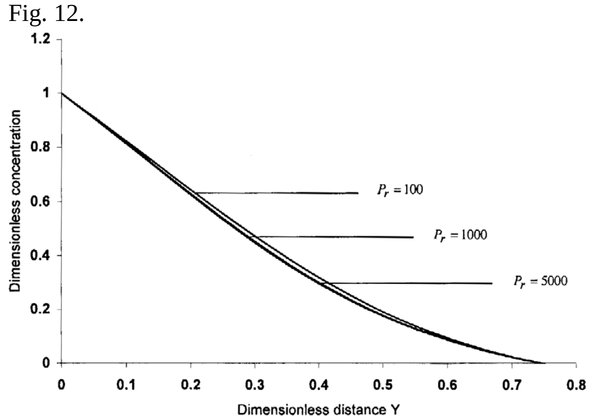



Fig. 12.




Figs. 10 – 12 show that the velocity increases with increasing $P_r$, while the temperature and concentration decrease with increasing $P_r$.

i. e.

$$a_0 + a_1 + 2a_2 v \leq 1,$$

and then

$$u\frac{\Delta t}{\Delta x} + |v|\frac{\Delta t}{\Delta y} + 2v\frac{\Delta t}{(\Delta y)^2} \leq 1. \quad (27)$$

Fig. 13.

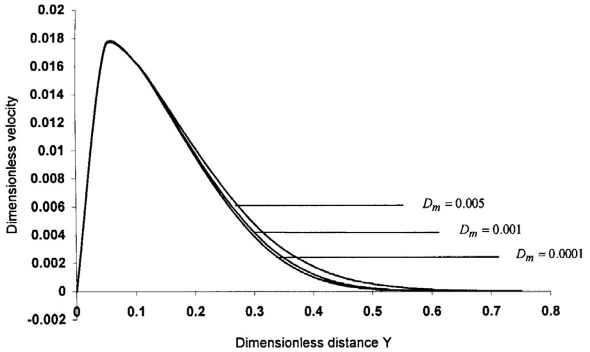



Fig. 14.

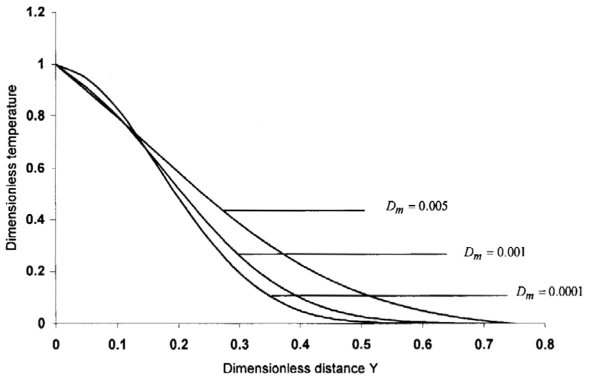



Fig. 15.

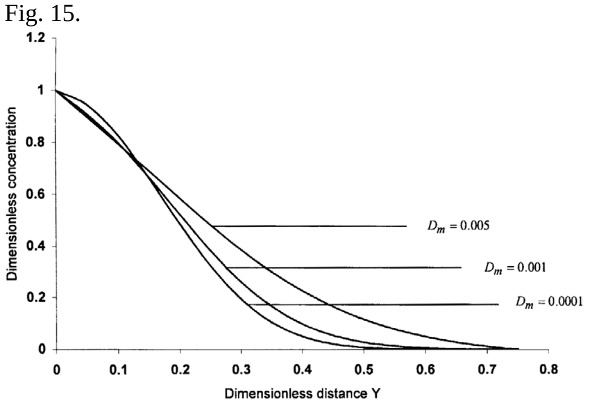



Fig. 16.

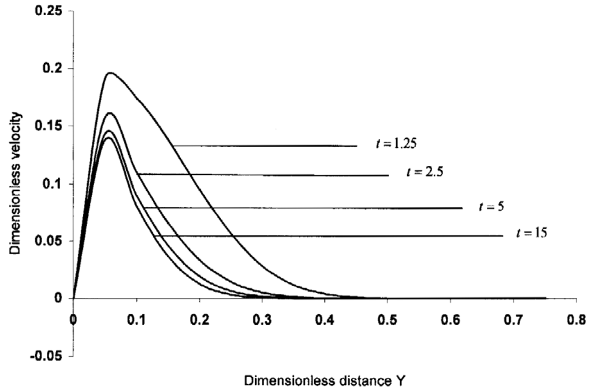



Fig. 17.

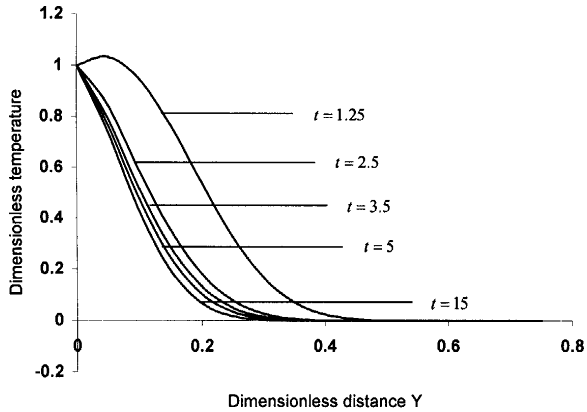
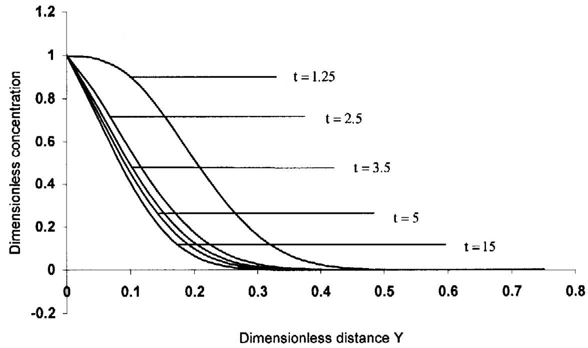



Fig. 18.




Figs. 13 – 18 show that the velocity, temperature and concentration increase with increasing $D_m$ and decrease with increasing $t$.

Similarly, to satisfy $|\lambda_2| \le 1$ and $|\lambda_3| \le 1$ we can put the stability conditions in the form

$$u\frac{\Delta t}{\Delta x} + |v|\frac{\Delta t}{\Delta y} + 2\alpha\frac{\Delta t}{(\Delta y)^2} + \frac{1}{2}Q \le 1, \tag{28}$$

and

$$u\frac{\Delta t}{\Delta x} + |v|\frac{\Delta t}{\Delta y} + 2D_m\frac{\Delta t}{(\Delta y)^2} \le 1. \tag{29}$$

Combining (27) – (29), we obtain the following condi-

Fig. 19.

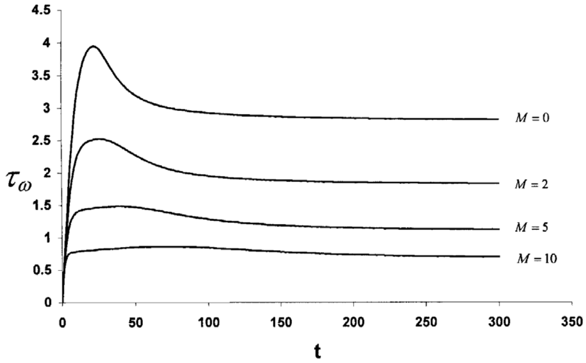



Fig. 20.

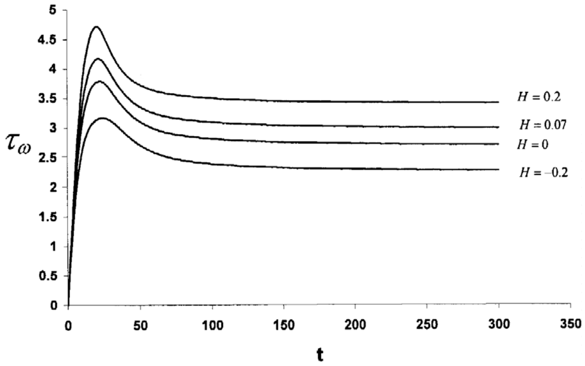



Fig. 21.

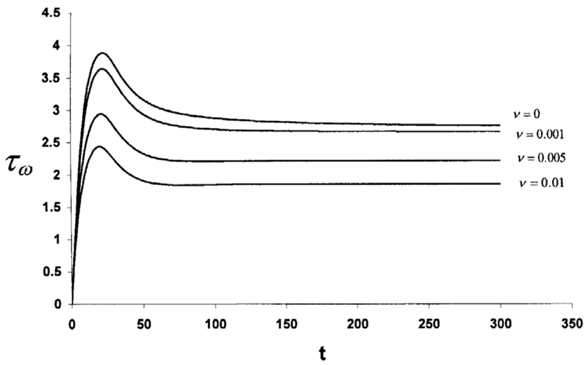



Fig. 22.

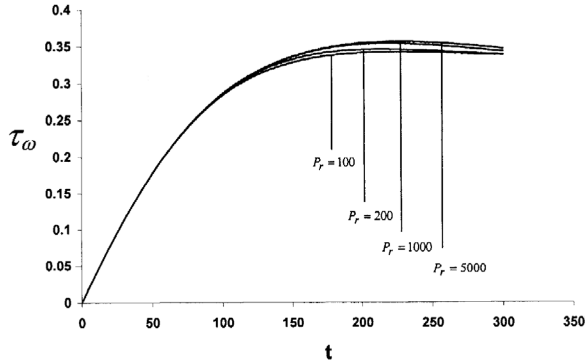
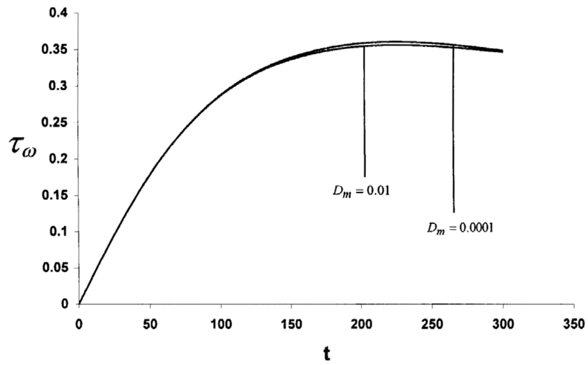



Fig. 23.




Figs. 19 – 23 show that the dimensionless skin friction $\tau_\omega$ decreases with increasing $M$, $\nu$ and $D_m$ and increases with increasing $H$, and $P_r$.

tion of stability:

$$u\frac{\Delta t}{\Delta x}+|v|\frac{\Delta t}{\Delta y}+\frac{\Delta t}{(\Delta y)^2}\left[\nu+\alpha+D_m\right]+Q\leq 1. \quad (30)$$

## 4. Method of Solution

The choice of mesh lengths, time-step and parameters becomes convenient when the finite difference scheme is considered in terms of variables with proper dimensions and hence the difference Eqs. (12) – (15) and the stability condition (30) are written as

$$\frac{u'_{i,j}-u'_{i-1,j}}{\Delta x}+\frac{v'_{i,j}-v'_{i,j-1}}{\Delta y}=0, \quad \text{(I)}$$

$$\frac{u'_{i,j}-u_{i,j}}{\Delta t}+u_{i,j}\frac{u_{i,j}-u_{i-1,j}}{\Delta x}+v_{i,j}\frac{u_{i,j+1}-u_{i,j}}{\Delta y} = \nu\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{(\Delta y)^2}-\frac{\sigma_0 B_0^2}{\rho}u_{i,j} + g\beta_0(T_w-T_\infty)\tau'_{i,j}+g\beta_1(C_w-C_\infty)\theta'_{i,j}, \quad \text{(II)}$$

Fig. 24.

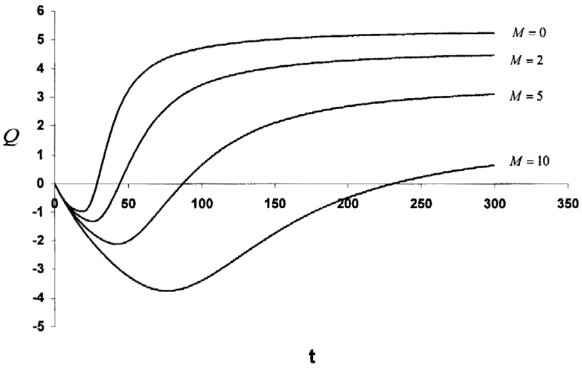



Fig. 27.

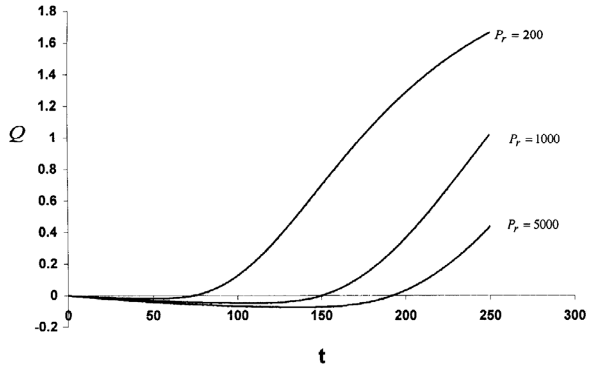



Fig. 25.

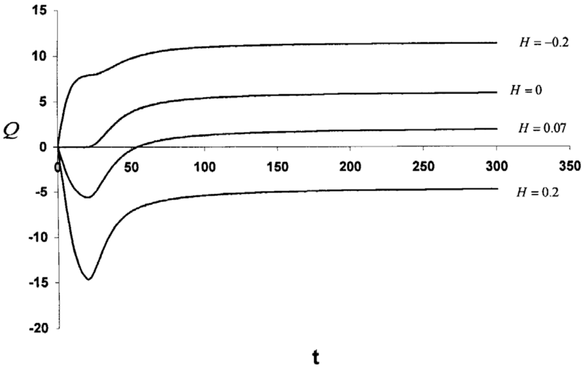



Fig. 28.

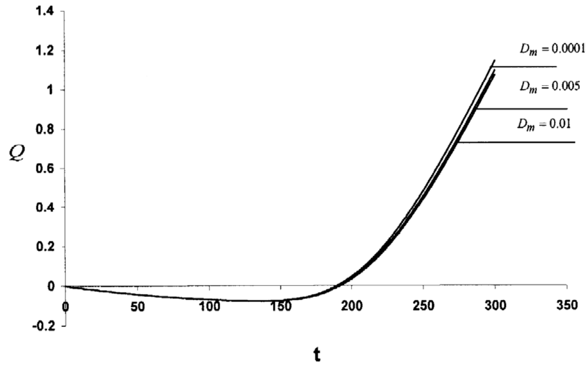
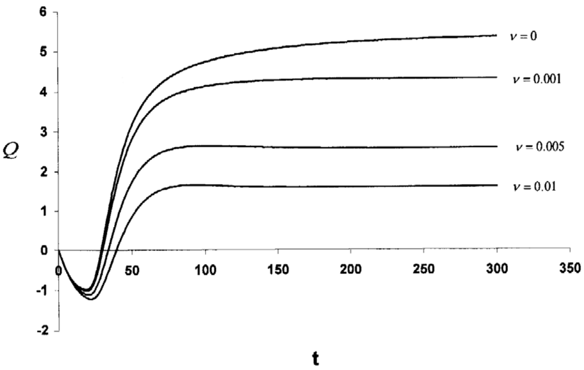



Fig. 26.




Figs. 24 – 28 show that the dimensionless local wall heat flux $Q$ decreases with increasing $M$, $H$, $\nu$, $P_r$ and $D_m$.

$$\frac{\tau'_{i,j}-\tau_{i,j}}{\Delta t}+u_{i,j}\frac{\tau_{i,j}-\tau_{i-1,j}}{\Delta x}+v_{i,j}\frac{\tau_{i,j+1}-\tau_{i,j}}{\Delta y} = \alpha\frac{\tau_{i,j+1}-2\tau_{i,j}+\tau_{i,j-1}}{(\Delta y)^2}+Q(T_w-T_\infty)\tau'_{i,j}, \quad \text{(III)}$$

$$\frac{\theta'_{i,j}-\theta_{i,j}}{\Delta t}+u_{i,j}\frac{\theta_{i,j}-\theta_{i-1,j}}{\Delta x}+v_{i,j}\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta y} = D_m\frac{\theta_{i,j+1}-2\theta_{i,j}+\theta_{i,j-1}}{(\Delta y)^2}, \quad \text{(IV)}$$

and

$$u\frac{\Delta t}{\Delta x}+|v|\frac{\Delta t}{\Delta y}+\frac{\Delta t}{(\Delta y)^2}(\nu+\alpha+D_m)+Q\le 1 \quad \text{(V)}$$

with the new initial and boundary conditions

$$t\le 0: u_{i,j}=0,\, v_{i,j}=0,\, \tau_{i,j}=0,\, \theta_{i,j}=0,$$

$$t>0: u_{i,j}=0,\, v_{i,j}=0,\, \tau_{i,j}=0,\, \theta_{i,j}=0 \text{ at } x=0,$$

$$u_{i,j}=0,\, v_{i,j}=0,\, \tau_{i,j}=1,\, \theta_{i,j}=1 \text{ at } y=0,$$

Fig. 29.

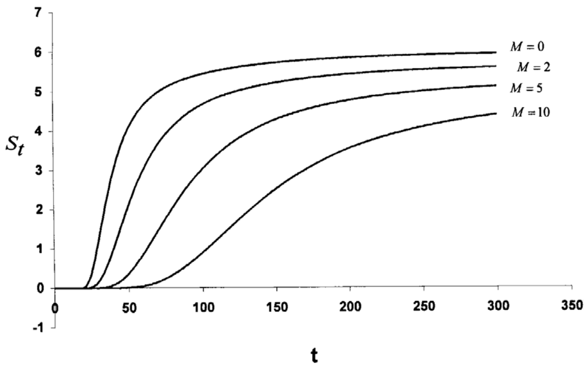



Fig. 32.

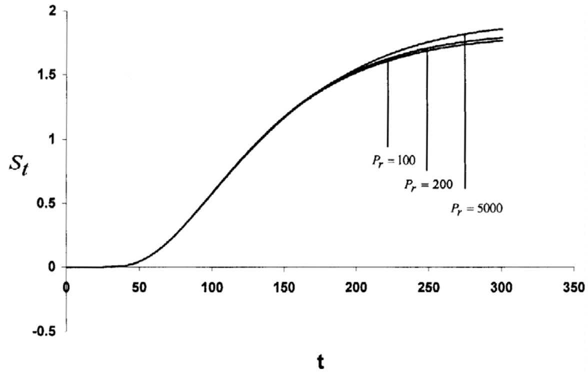



Fig. 30.

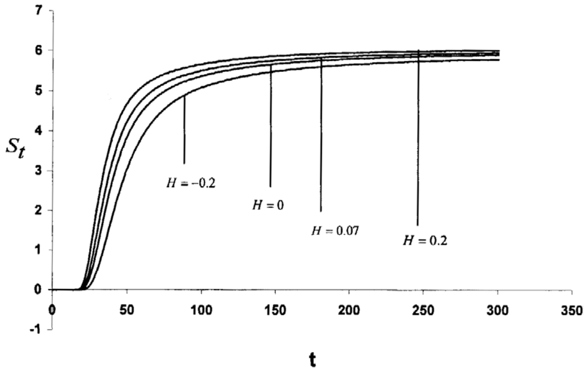



Fig. 33.

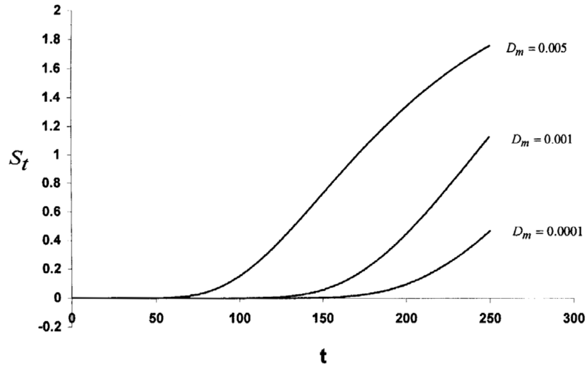
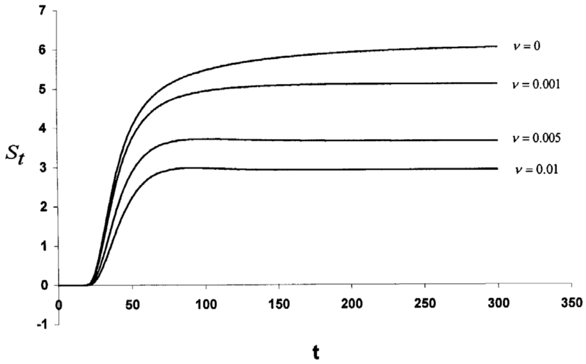



Fig. 31.




Figs. 29 – 33 show that the dimensionless rate of mass transfer $S_t$ decreases with increasing $M$ and $\nu$ and increases with increasing $H$, $P_r$ and $D_m$.

$$u_{i,j} = 0,\ \tau_{i,j} \to 0,\ \theta_{i,j} \to 0 \text{ as } y \to \infty. \qquad \text{(VI)}$$

where the primed variables indicate the value of the variable at a new time, and $(i, j)$ represent grid points.

## 5. Results and Discussion

In order to get an insight into the physical situation of the problem, we have computed the velocity, temperature and concentration for different values of various parameters occurring in the problem.

The numerical calculations were performed by taking $12 \times 15$ meshes over the region, $x = 36.6$ cm and $y = 22.9$ cm. The time-steps were uniform and given by $\Delta t = 0.05$ s, while the parameters assumed the values

$$g = 975 \text{ cm s}^{-2}, \quad T_w - T_\infty = -13.9\ ^\circ\text{C},$$
$$C_w - C_\infty = 5.10^{-3} \text{ mol cm}^{-3},$$
$$\rho = 1 \text{ g cm}^{-3}.$$

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